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Nonlinear Finite Element Modelling and Parametric Study of Prestressed Steel Box Girder

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Abstract

The prestressing technique is easy to apply and is generally used to strengthen steel bridges and controls their gross deflection. ANSYS has been used to establish a numerical model for the mechanical behavior of a steel box girder and prestressed by external tendons. In this paper, steel plate girders with and without strengthening technique was tested to assess the effectiveness of this technology. The results showed that prestressing improves the mechanical behavior of a girder and that its effect is proportional to magnitude of the applied external load. The results of the numerical model showed good agreement with the experimental data. A full-course simulation was conducted with ANSYS for a parametric study to analyze the influence of prestressing force magnitude, span-height ratio, and tendon configurations in increasing the effectiveness of prestressed technique.

Key words-*Prestressing technique; steel plate girder; Dual nonlinear problem; ANSYS; tendon configuration; deviator.*

I. INTRODUCTION

The external prestressing technique, which was initially developed for bridges, is now becoming more popular for bridge construction and strengthening or rehabilitating existing structures [1], [2]. This technique can be defined as subjecting the structural member to loads in the reverse direction of the exterior load. The analysis of girders prestressed with tendons, whether internal or external, offers several computational difficulties and challenges for proper modeling. In an externally prestressed system, the tendons are fixed outside the structural member and attached to the girder by anchors at the two ends and deviators along the girder span. The tendons' stress at any load level relies on the global deformation of the structure [3], [4]. Depending on the type of structure and its future working conditions, the prestressing technique can be applied during the erection of new structures such as girders, frames, trusses, buildings, arches, towers, and bridges. However, this technique can be more effective if the prestressing force is applied in more than one step so that the prestressing and required loading are achieved in a few cycles [5]. Prestressing may in turn be applied to any material, particularly to steel construction. Strengthening a steel structure with the prestressing technique creates artificial stress opposite to stresses due to external imposed loadings (i.e., dead and live loads). The finite-element analysis software ANSYS is a large general-purpose package with strong pre-and post-processing capabilities. It provides a wealth of element types for nonlinear analysis and can potentially be applied to steel girders. Traditional numerical methods for nonlinear steel girders have several disadvantages that make it difficult for them to meet the actual requirements. In particular, computational analysis becomes very difficult for girders in the plastic stage [6]. The basic concept of the finite-element method is analyzing a continuous body that is virtually divided into a combination of a finite number of elements, which are referred to be as discrete. Because the elements can be coupled and combined in different ways and the elements can have different shapes, they can be used to

model complex geometry for the solution domain. The finite-element method is for numerical analysis. Another important feature is the use of an approximate function. Each element is assumed to represent the whole existence of discrete points within the domain that are searched for to solve the unknown field functions. An approximate function normally expresses values of an unknown field function or its derivative and uses interpolation to determine each node element. Thus, the unknown field function or its derivative value at each node becomes the new unknown quantity, so a continuous unconstrained issue can be transformed into a discrete constrained issue. By solving these unknowns, we can interpolate each field element function to obtain approximate solutions for the entire solution domain. Increasing the number of cells and thus reducing the cell size increases and improves the accuracy of the interpolation function and thus the degree of validity of the approximate solutions. If the element meets the convergence requirements, the final approximate solution converges to the exact solution [7], [8], [9].

II. MATERIAL AND METHODS

A. Numerical Model

A steel box girder was analyzed twice: once with prestressing and the other without prestressing. A test was performed to check the improvement in the load-carrying capacity with the applied prestressed technique. The actual size, material properties, and geometry of the girder inserted into the ANSYS program were similar to those of the steel girders tested in the experiments. The SHELL63 and LINK180 elements were applied to simulating the steel box girder and tendons, respectively. Since the prestressed tendon is an elongated material, it can ignore its transverse shear action and treat it as a linear rod element. The steel was regarded as an ideal elastoplastic body.

1) *Dual nonlinear problem*: Nonlinear problem structures, including geometric non-linearity, material non-linearity, and the non-linear state. For geometric non-linearity, the

displacement-strain relationship of the structure is non-linear. This can be divided into small strain-large displacement and large displacement-large strain (i.e., finite displacement and strain). For material non-linearity, the stress-strain relationship is non-linear. In double non-linear problems, both geometric and material non-linearly must be considered. That is, not only the displacement-strain relationship but also the stress-strain relationship is non-linear. The processes for non-linear and linear finite-element analyses are basically the same and involve three tasks: (1) select the type of logical element or discrete structure; (2) select the strain and displacement mode to obtain the tangent stiffness matrix; and, (3) together with the total stiffness matrix, establish incremental equations to determine the structural equilibrium and numerically solve the problem. The difference between linear and non-linear finite-element analyses is in the formation of the element stiffness matrix after the discrete structural element is selected. For non-linear finite-element analysis of a structure, the following should be considered: (1) When only geometric non-linearity is considered to calculate the strain-displacement matrix, the response should consider the displacement of higher derivative terms. Changes in the element volume should be considered at all points. (2) When only material non-linearity is considered, a non-linear constitutive relationship should be used. (3) When both geometric and materials non-linearly are considered (i.e., a double nonlinear problem), the two nonlinear effects should be considered together [10], [11], [12], [13], [14], [15], [16], [17].

2) *Establishment of the nonlinear finite element equation:* To solve dual nonlinear problems, the geometry of the structure and material non-linearity both need to be considered. Both the displacement-strain and stress-strain relationships are nonlinear. The nonlinear finite-element equilibrium equation is established according to the virtual displacement principle [18], [19], [20], [21], [22], [23], [24], [25], [26], [27]:

$$\{\psi(\delta)\} = \int [B^-]^T \{\sigma\} dV - \{P\} = 0 \quad (1)$$

Where: $\{\psi\}$ represents the node generalized forces, generalized force vector, and various loads. Eq. 1 is applicable to large and small displacements. For a large displacement, the strain-displacement relationship is nonlinear. The matrix $[B^-]$, $\{\sigma\}$ is a function and can be written as:

$$[B^-] = [B_0] + [B_L] \quad (2)$$

Where: $[B_0]$ item matrix linear strain and nonlinear deformation. The stress-strain relationship for nonlinear materials is:

$$[\sigma] = [D]_{\varepsilon p} \{\Delta \varepsilon\} = ([D] - [D]_p) \{\Delta \varepsilon\} \quad (3)$$

where $[D]_{\varepsilon p}$ the elastoplastic matrix. To solve nonlinear problems with finite-element equilibrium equations, the Newton-Raphson raw method is applied to create the incremental relationship between $\{d\delta\}$ and $\{d\psi\}$:

$$\{d\psi\} = ([k_0] + [k_L] + [k_\sigma] - [k_R]) \{d\delta\} = [k_T] \{d\delta\} \quad (4)$$

Fig. 1. Finite element model of prestressed simply supported steel box girder.

where $[k_T] = [k_0] + [k_L] + [k_\sigma] - [k_R]$. Here, $[k_0]$ is a small displacement linear stiffness matrix, $[k_L]$ is a large displacement stiffness matrix, $[k_\sigma]$ is the geometric stiffness matrix, $[k_R]$ is the load correction matrix, and $[k_T]$ is the tangent stiffness matrix for the double nonlinear structural problem.

3) Geometric Model and Mesh:

a) *Create a geometric model:* ANSYS provides two methods for creating finite-element model, namely, direct and geometric mesh modeling methods. In this study, the finite-element model is established by a direct method. The longitudinal direction of the model is defined as Z-coordinate, the horizontal direction as X-coordinate and vertical as Y-coordinate. The steel girder and the tendon are determined from different element types. The geometric model is established and the element is divided and meshed. The connection between the steel girder and tendon is done by the method of node damage. In the process of the model due to the shorter length of the tendon, so ignoring the friction effect between the girder and tendon. It is considered that the tendon can slip freely along the length of the girder in Z direction, and the connection nodes between the steel girder and tendon are merged in the X and Y directions, this point is called deviator, thus realizing the interaction between the two.

b) *Divide and Mashing:* In the ANSYS, the mesh is divided into several categories, such as free division, mapping division, sweeping division and so on. In this model, the girder and the tendon are meshed by the method of free division, and the regular element shape is obtained to improve the analysis accuracy and calculation speed. There are 5492 elements in this model, in which the number of SHELL63 elements is 5476, and the number of LINK180 elements is 16. The mesh model of the prestressed simply supported steel box girder is shown in Fig. 1.

4) Applied Load:

a) *Displacement constraint:* The displacement constraints are applied to the numerical model. That is, the boundary condition is defined. One end of the girder is completely bound by the freedom of the three directions X, Y and Z, while the freedom of X and Y direction at the other end is restrained, and the Z direction is free to slip.

b) *Prestressing force*: There are three types of prestressing force methods in ANSYS: Equivalent load method, Cooling method and Initial strain method. The Equivalent load method basically is to replace the steel tendon with the form of load. The advantages of this method are: modeling is easy, and can be modeled directly without considering the location of the tendon. Undoubtedly, the mesh is also simple, and the overall effect of the structure under the action of prestressing can be quickly mastered. But its shortcomings include: the equivalent load method does not consider the direction of force and reinforcement on the girder; the tensioning process cannot be simulated because of inconveniences to simulate the details. For instance, the impact of the location force on the structure is different and should be simulated. Cooling method is relatively simple, since only the definition of several elements and real constants are required, the loss of the prestressing force can be simulated, and the solution is accurate. When the initial strain method is used to simulate different stresses in the tendon, the real constants of each element are unequal and the workload is larger. Therefore, in comparison, the cooling method is adopted in this model.

According to the thermal expansion coefficient and elastic modulus of tendon, the effect of prestressing force is converted into the equivalent temperature force according to the formula 5, and it is applied across a temperature force to the tendon to realize the prestressing effect.

$$\Delta T = -\frac{\sigma_{ye}}{\alpha E} \quad (5)$$

Where ΔT is Equivalent temperature force, σ_{ye} is Prestressing force, α is linear expansion coefficient of tendon, E is tendon Elastic modulus. Prestressed steel structures have the same prestressing loss as concrete structures. There are many factors that cause prestressing force loss, such as the friction between the tendon and girder at the deviator points, the slipping of the anchors, and the relaxation of the tendon. For prestressing loss calculation, the existing codes are calculated by sub item, and then the total loss is calculated after superposition. The prestressing loss of steel structure consists of two parts namely, immediate loss and long-term loss. The instantaneous losses include the loss of friction, and anchorage (including the anchor deformation, and tendon slip) and the elastic compression loss of the steel girder by the pre-tensioning. Long-term losses are mainly due to the tendon relaxation loss, which take a long time to complete. The creep of steel is very small, and the effect of the prestressing loss is negligible.

c) *External load*: In order to study the stress on condition of ultimate failure of a prestressed steel box girder, the model continues to load based on the load test until and the girder is destroyed. Consequently, multiple load steps are placed in the model, and the sub-step is loaded.

5) *Solution settings*: Before analyzing the finite-element model, you need to set the solution control options, including the definition of analysis types, set the analysis of common options. In this study, the static analysis of a prestressed simply supported steel box girder is carried out. The complete Newton

Fig. 2. Model details of steel box girder prestressed with external tendons.

Raphson method is used to solve the nonlinear problem. The following options are set:

- 1) (nlgeom, on) is used to consider the effect of large deformation or strain on the element, and considering geometric non-linearity.
- 2) (pred, on) Defining predictions in a nonlinear analysis usually helps to reduce the number of required equilibrium iterations.
- 3) (lnsrch, on) using linear search methods usually contributes to the convergence of results.

For convergence criteria, the regular control force, displacement, and moments are usually used. In this study, both the force and displacement are adopted, and the convergence accuracy is set to 50%.

B. Model validation test

1) *Experimental mode*: To facilitate analysis and comparison, a simply supported steel box girder (with and without prestressed girders) was tested in this study. The geometric dimensions of the model are 1/10 of the actual bridge size (bridge span is 40 m. Height is 1.25 m. Width is 7.2 m). The test girder is a single box girder welded to a Q235 steel plate, and its span is 3.99 m. We consider a high span ratio of 1/32 and two steel tendons, each consisting of seven high-strength strands with a diameter of $\varnothing = 5\text{mm}$, as shown in Fig. 2(a, b & c). The steel tendons pass through a set of rib stiffeners at deviator points, and they are anchored at two ends of the girder. TABLE I presents the dimensions and material properties of the girder and tendon. The layout of test points on the girder is shown in Fig. 2.

2) *Comparison between ANSYS calculation results and experimental data*: The static loading test was performed

TABLE I
DIMENSIONS AND MATERIAL PROPERTIES OF STEEL GIRDER AND TENDONS.

Symbol	Name	Dimensions (mm)	Quantity	Material	Unit weight (kg)	Yield strength (MPa) f_y	Elastic modulus E(MPa)	Poisson ratio	Linear expansion coefficient (α)
H_1	Top flange	$272 \times 4 \times 4000$	1	Q235	34.16	235	2.06×10^5	0.3	-
H_2	Bottom flange	$160 \times 4 \times 4000$	1	Q235	20.10	235	2.06×10^5	0.3	-
H_3	Web	$116 \times 4 \times 4000$	2	Q235	14.57	235	2.06×10^5	0.3	-
H_4	Stiffener A	$(20 + 50) \times 4 \times 74$	8	Q235	0.1	235	2.06×10^5	0.3	-
H_5	Stiffener B	$(20 + 116) \times 4 \times 74$	9	Q235	0.17	235	2.06×10^5	0.3	-
H_6	Diaphragm	$116 \times 4 \times 116$	9	Q235	0.42	235	2.06×10^5	0.3	-
H_7	Bearing Plate	$40 \times 4 \times 40$	4	Q235	0.06	235	2.06×10^5	0.3	-
H_8	Steel tendon	7 ϕ 5 strand $\times$ 6000 length	2	$f_{pk} = 1860$ MPa (low relaxation)	-	-	1.95×10^5	0.3	1.2×10^{-5}

on the prestressed and control girders. The deflections and strain values of the top flange girder at mid-span from ANSYS and the experiment model were calculated, as given in TABLE II and III. TABLE IV presents the tension increment force (h). Experiment and ANSYS considered steel girders with the same dimensions and with and without the strengthening technique. Comparing the results of deflection in the mid-span of the girder from TABLE II, and TABLE III using experiment analysis, we found that after applying 2.5 kN/m load to the control girder, there was a deflection value of 5.9 mm. In the case of applying the same external load on the prestressed girder, the deflection value was 4.94 mm. This showed a reduction of about 16% because of applying strengthening technique. Furthermore, when ANSYS was used for the same load, a deflection reduction of about 15% was recorded for the prestressed girder. This reduction applies to a prestressed girder subjected to various loads. In addition, in case of the strain as shown in TABLE II and II for experiment analysis, the top flange strain of control and prestressed girder at mid-span duo to applying external load 2.5 kN/m is 219.3 $\mu\epsilon$, and 194.8 $\mu\epsilon$, respectively. This was a strain reduction of about 11% due to applying strengthening technique. When ANSYS analysis was used, the strain reduction was about 21%. Applying the prestressing method led to a stiffer girder with reduced deflection and strain. The comparison between the experimental and finite-element calculation showed a relatively small error, which suggests that their results were in good agreement. Thus, it is practical for ANSYS to establish the finite-element model, and the program can be used in a further analysis of prestressed steel box girders under static loads.

TABLE II
EXPERIMENTAL AND FINITE-ELEMENT CALCULATION VALUES FOR THE CONTROL STEEL GIRDER

Load	Span deflection (mm)			Top flange strain ($\mu\epsilon$)		
	Test	ANSYS	Error %	Test	ANSYS	Error %
2.5 kN/m	5.9	5.75	2.5	219.3	229.1	5.6
10 kN	9.4	9.7	3.2	440.2	445	1.8

TABLE III
EXPERIMENTAL AND FINITE-ELEMENT CALCULATION VALUES FOR THE PRESTRESSED STEEL GIRDER.

Load	Span deflection (mm)			Top flange strain ($\mu\epsilon$)		
	Test	ANSYS	Error %	Test	ANSYS	Error %
2.5	4.94	4.9	0.8	194.8	179.9	7.6
5	11.1	10.8	2.7	410.7	385.46	6.1
10	8.85	8.4	5.1	392.8	366	6.8
20 kN	18.2	17.67	2.9	814.2	764.1	6.2

TABLE IV
EXPERIMENTAL AND FINITE-ELEMENT CALCULATION VALUES FOR THE TENSION INCREMENT FORCE.

Tension increment force h (kN)							
Load (kN/m)	Test			Load (kN)	Test		
	Test	ANSYS	Error %		Test	ANSYS	Error %
1.25	1.91	1.97	3.1	5	2.89	3.05	5.5
2.5	3.9	3.95	1.3	10	6.16	6.09	1.1
3.75	5.89	5.92	0.5	15	9.09	9.14	0.6
5	7.7	7.89	2.5	20	12.14	12.18	0.3

III. MECHANICAL PERFORMANCE ANALYSIS OF PRESTRESSED STEEL BOX GIRDER

A. Load-displacement curve analysis

A steel box girder was strengthened after a pre-tensioning load of 82.5 kN was applied. Fig. 3 presents the load-displacement curves of the prestressed and control girders. At the beginning of the load phase, the displacement showed a linear relationship with an increasing load. The curve shows an obvious turning point that corresponds to the ultimate load. According to the ANSYS model, the maximum loads of the control and prestressed girders were 79 and 92 kN/m, respectively. This demonstrates the efficiency of the prestressing technique. The ultimate load increased about 14% under the same conditions, while the mid-span displacement of the prestressed girder was less than that for the control girder. Applying a relatively large external force clearly produced a significant difference. This suggests that prestressing a steel

Fig. 3. Load-displacement curves for steel box girders.

girder can improve the bearing capacity and bending stiffness. Fig. 3 shows that, when the steel box girder was under the ultimate load, adding to the load significantly increased the displacement for both girders. However, the displacement of the prestressed steel box girder grew slower than that for the control girder. When plastic hinges appear at the mid-span, the control girder has no extra constraint, so the geometry becomes variable. In contrast, the stress for the prestressed girder had not yet reached its yield strength, so the girder was still in a flexible working phase. The prestressed steel girder only became a statically determinate structure.

B. Normal Stress Cloud Analysis

Fig. 4 shows the stress cloud of a steel box girder with a prestressed force of 82.5 kN. The stress curve of a steel box girder is in an inverted arch state from which the external load is produced, when the cross-section of the steel box girder is under pre-tensioning force, the top flange girder produces tension stress, and the bottom flange produces compressive stress. During the loading stage, when the load increases up to 2.75 kN / m, the stress cloud of prestressed girder is shown in Fig. 5. At this point, the top flange girder stress across the middle section gradually changes from the tension stress to near zero stress state, and the bottom flange stress is progressively reduced. With the increase of external load, the reverse arch caused by the pre-tensioning force gradually disappears, and the top flange girder changes from the zero-stress to the compressive stress state. When the load is 10.5 kN, the stress diagram shown in Fig. 6, it is obvious from this figure, the top flange compressive stress imperceptibly increased, and the bottom flange changed from the compressive stress to near zero stress state, continued increase in the load will change

Fig. 4. Stress Cloud Diagram of Steel Box Girder in Prestressed Stage.

Fig. 5. Stress Cloud diagram of prestressed steel box girder under the external load of 2.75kN/m.

Fig. 6. Stress Cloud diagram of prestressed steel box girder under the external load of 10.5kN/m.

the stress state to the tension stress. The stress cloud of a prestressed girder under ultimate load is shown in Fig. 7. At this point, the part of girder cross-section changes to the plastic stage. If you continue increasing the external load, the prestressed girder will produce a small deformation (stability loss and damage). As shown in Fig. 8, the prestressed girder has a large twist deformation close to the cross section.

Fig. 7. Stress Cloud diagram of prestressed steel box girder under the ultimate extreme load.

Fig. 8. Stress Cloud diagram of prestressed steel box girder under the stability loss and damage.

C. Sectional strain analysis

Based on the analysis of the cross-sectional strain under various external loads on mid-span girder, the prestressed girder mid-section showed great linear changes with different strain actions under a uniformly distributed load, as shown in Fig. 9. The girder section deformation remained uniform in the elastic phase. Even when part of the section reached the plastic stage, the cross-sectional deformation still same. This is in line with the flat section assumption.

D. Nonlinear factor analysis

Girder stress performance is affected by various nonlinear factors. In this study, the analysis effects of material and geometrical non-linearity are considered. In order to analyze the girder influence of nonlinear factors, the numerical calculation is divided into three cases: first case is taken as linear elastic material; the second is to consider only nonlinear geometric; third case is to consider both are non-linearity (That is the double non-linearity). The results can be expressed by the load-displacement curve at mid-span girder, as shown in Fig. 10. As it can be seen from Fig. 10 that when the material is linear elastic, the load-displacement curve is straight and exhibits a clear proportional relationship. When considering nonlinear geometric, the load-displacement curve is growing

Fig. 9. High variation in the cross-sectional strain along the height of the prestressed girder.

linearly at the beginning of load stage, and coincides with the straight line when the material is linearly elastic, which indicates that the girder is in the elastic stage during this time. With increasing load, the load-displacement curve is no longer growing as linearly, the displacement growth rate is greater than the loads, and finally the girder deformation is too large and caused damage. When considering double non-linearity, the load-displacement curve is almost coincident with the nonlinear geometrical only, which indicates that the nonlinear material has a little effect on the mechanical behavior of a prestressed girder. In view of the above analysis, it is clear that the mechanical properties of a prestressed girder studied in this paper are greatly affected by the nonlinear geometric compared to the material, mainly because the prestressed girder has good plasticity.

IV. PARAMETRIC STUDY

In this paper, the finite-element model built by ANSYS program is used to study the parametric effects on the prestressed technique. Which has studied the effect of prestressing force magnitude, tendon configurations, and girder span-height ratio on mid-span deflection, top and bottom flange stress and tension increment force of tendon.

A. Effect of prestressing force magnitude

The finite-element model of the prestressed steel box girder is used to analyze the Effect of prestressing force magnitude on the mechanical behavior of a steel girder and external tendon. The pre-tensioning force H is 40kN, 60kN, 80kN, 100kN and 120kN. Tendon area is 139mm².

1) *Effect of prestressing force magnitude on mid-span deflection:* Fig. 11 present load-displacement curve, the prestressed girder produces a certain deflection in reverse direction to the external load. The deflection value increases with the increment of tension force. Exerting different magnitude of

Fig. 10. Effect of different nonlinear factors on the load-displacement curve at mid-span girder.

Fig. 11. Load-displacement curve under the effect of prestressing force magnitude.

prestressing force under the same external load has a negligible effect on the mid-span deflection, which results to insignificant effect on the prestressed girder deformation.

2) *Effect of prestressing force magnitude on top and bottom flanges stress:* Fig. 12 depicts load-stress curves, as it can be seen with increment in the tension force, stress values of the top and bottom flange reduce proportionally. However, the tensioning force has a certain limit depending on the tendon strength, which must not exceed the allowable design value.

Fig. 12. Stress-load curve of top and bottom flange girder under the effect of prestress force magnitude.

Fig. 13. Load-Increment Tension Force Curve under the effect of prestressing force magnitude.

Also, as it can be seen at the same external load values, the prestressing force increases from 40kN to 120kN, causing stress reduction in the top flange by about 39MPa, while the bottom flange stress reduces by about 156.6 MPa. Therefore, the effect of prestressing force magnitude on bottom flange stress is greater than top flange stress.

3) *Effect of prestressing force magnitude on tendon tension increment force:* Tendon tension increment forces were plotted, as shown in Fig. 13. The magnitude of the prestressing force at the same load values has a negligible effect on the tension increment of tendon. The increments of tension force and external load showed a linear trend.

B. Effect of tendon configurations

The mechanical behavior of a prestressed girder depends largely on the reasonable arrangement of the steel tendon. In this study, the effect of four types of tendon configurations,

Fig. 14. Elevated view of steel box girders with different tendon configurations.

such as the straight linear tendon without deviators (tendon configuration 1), draped tendon with single deviator (tendon configuration 2), draped tendon with double deviators; the space between them is the $\frac{1}{4}$ span girder (tendon configuration 3), and curve tendon with 7th deviators (tendon configuration 4). The arrangements of the tendon configurations were used as shown in Fig. 14. The prestressing tension force is 82.5 kN, and the tendon area is 139 mm.

1) *The effect of tendon configurations on mid-span deflection:* The load-displacement curve is shown in Fig. 15. The prestressed girder deflection values at the prestressing stage of tendon configurations 2 and 3 are nearly close. Tendon configuration 1 is much smaller than that of the other three (tendon configurations 2, 3, 4) under tension force. This is because the generation effect of prestressing force at both girder ends is very small. The tendon configuration 1 has a negligible effect than others in the mid-span deflection values produced by the same external load. Therefore, the deformation of the tendon configuration 1 of a prestressed girder is relatively large and unfavorable.

2) *Effect of tendon configurations on stress of top and bottom flange girder:* Fig. 16 shows the stress distributions of girder flanges along the prestressed girder length at pre-stressing stage. The top and bottom flange stresses were for the tendon configuration 2 produced the maximum tension and compression stress respectively. The stress at this stage is controlled by the bending point stress of tendon configuration 3, which is tension or compression stress section from the end to the $L/4$ girder, at this section the different tendon configuration shows a great stress effect than other sections. Obviously, the stress values from $3L/8$ to the $L/2$ section are close to each other at this stage. The girder flanges stress value from tendon configuration 1 is compressed under the prestressing force, and the stress value of all tendon configurations is not much different. In the $L/4$ stress section, the compressive stress of top flange is partially small, and bottom flange is slightly larger than the other sections. From the positioning, the Tension and Anchor point of view for tendon configuration 1 is the simplest and convenient. However, for a simply supported girder, the

Fig. 15. Load-displacement curve under effect of tendon configurations.

Fig. 16. Stress-Load curve of top and bottom flange girder under the effect of tendon configurations in the prestressing stage.

straight tendon is not favorable, because the moment force generated near the supports is too small, but when applying the prestressed technique, it can lead to adverse stress state.

The stress distribution curves for top and bottom flange along the prestressed girder length, under the load action of 5 kN/m, 10 kN/m, 15 kN/m and 20 kN/m are shown in Fig. 17, 17, 19, 20, respectively. Through the comparative analysis of the following groups of graphs, at the same load value, can be seen that the top and bottom flanges stress distribution of tendon configuration 3 from the end to the $1/4$ girder section, the effect of draped tendon is better than the straight. Therefore, the tendon configuration 1 is the most unfavorable. Furthermore, the equivalent load produced by the tendon configuration 3 is similar to tendon configuration 4 on a prestressed steel box girder.

3) *Effect of tendon configurations on tendon tension increment force:* Fig. 21 shows the load-tension force curve, from the tendon configuration 1, the effect of tendon configuration on tension increment force of tendon is less significant than other. The tension increment force from tendon configuration 1 is smaller than the other. Obviously, with increased external load, the difference for tension increment force automatically increases.

Fig. 17. Stress-Load curve of top & bottom flange girder under the effect of tendon configurations at 5 kN/m.

Fig. 18. Stress-Load curve of top & bottom flange girder under the effect of tendon configurations at 10 kN/m.

Fig. 19. Stress-Load curve of top & bottom flange girder under the effect of tendon configurations at 15 kN/m.

Fig. 20. Stress-Load curve of top & bottom flange girder under the effect of tendon configurations at 20 kN/m.

C. The effect of span-height ratio on the prestressed steel box girder

The girder finite-element model with curve tendon is used to analyze the effect of a span-height ratio for the girder mechanical behavior. The span-height ratio (L/h) of the girder were taken 28, 29, 30, 31, 32. Tension force is 82.5 kN.

1) *The effect of span-height ratio on mid-span deflection:* Fig. 22 shows load-displacement curve, with increased span-height ratio, the mid-span deflection caused by the external load is gradually increased, and the span-height ratio effect is more significant. When the span-height ratio increases from

Fig. 21. Load-Increment Tension Force Curve under the effect of tendon configurations.

Fig. 22. Load-displacement curve under the effect of span-height variation.

28 to 32, it generates mid-span deflection of about 70% by the same external load. It is obvious that the span-height ratio has a significant effect on the girder deformation. Therefore, the reasonable decrease of a span-height ratio is helpful to reduce the girder deformation.

2) *Effect of span-height ratio on top and bottom flange stress girder:* Fig. 23 illustrates the load-stress curves, where the same load value with increased span-height ratio, the stress

Fig. 23. Stress-Load curve of top & bottom flange girder under the effect of span-height ratio.

Fig. 24. Load-Increment Tension Force Curve under the effect of span-height ratio.

value of top and bottom flanges is increased. When the amount of load is small, the span-height ratio has an insignificant effect on stress girder. Therefore, in order to reduce the stresses of top and bottom flanges of a prestressed girder, the span-height ratio can be appropriately reduced.

3) *Effect of span-height ratio on tendon tension increment force:* The load-tension force curve has been adopted in Fig. 24. The tendon tension force is increased by the span-height ratio augmentation. Therefore, when the amount of load is large, the span-height ratio effect in tension force is more pronounced.

V. CONCLUSIONS

In this study, steel box girders were built and tested in order to verify the effectiveness of applying the prestressing technique to a simply supported steel box girder. Finite-element analysis was performed with ANSYS, and the material and geometric properties were similar to those of the experimental steel girders. The following conclusions were reached:

- 1) Applying the prestressed technique to the simply supported steel box girder improved their bearing capacity, mechanical stiffness, and limit load while decreasing the rate of displacement in both the numerical and experimental models. In this study, the girder ultimate load increased by about 14%, and the structure rigidity improved and reduced the structure deformation.
- 2) Undoubtedly, when the steel girder reaches the maximum load, the mid-span deflection will increase significantly as long as there is a small load, and the displacement of a prestressed girder increased slower than the girder without prestressing. The prestressing technique can also solve the limited height section problem for the steel bridges because it makes the girders light and improved their durability.
- 3) The mechanical properties of the prestressed girder studied in this paper are greatly affected by the nonlinear geometry, while the effect of the nonlinear material is small, which is mainly because the prestressed girder has good plasticity.
- 4) The prestressing force magnitude has a significant effect on stress value of the prestressed girder at the top and bottom flange and with increased pre-tension force, the stress values are all reduced proportionally. This indicates that with increment of tension force, the stress values at both flanges are significantly decreased, but the prestressing force has a certain limit, which depends on tendon strength, and should not exceed the allowable design value.
- 5) In the practical engineering project, the reasonable decrease of a span-height ratio is helpful to decrease the deformation of the girder and reduces the stress girder cross-section at the top and bottom flange.
- 6) Under the same external uniform load, the effect of straight tendon (tendon configuration 1) on deformation of prestressed girder is relatively large.

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